Phase 12 – Quantization & Coupling  
Part 5: Quantum ψ Phenomena – Fluctuations, Excitations, Entanglement, Vacuum Effects

ψ Quantum Fluctuations  
In the quantized ψ-field, fluctuations arise around the vacuum expectation value ⟨ψ⟩.  
Two-point correlation function (Wightman function):

Plain text:  
G⁺(x,y) = ⟨0 | ψ(x) ψ(y) | 0⟩

Vacuum variance in Fourier space:

Plain text:  
⟨|ψ̃(k)|²⟩ = ħ / (2 ωk), where ωk = sqrt(k² + mψ²)

This spectrum defines the quantum ψ noise background.

ψ Excitations (ψ-Quanta)  
ψ excitations are quanta analogous to particles. Creation/annihilation operators:

Plain text:  
ψ(x) = ∫ d³k / (2π)³ 1/sqrt(2ωk) [ a\_k e^(−ikx) + a†\_k e^(ikx) ]

ψ-quanta mediate forces and can form collective excitations in ψ backgrounds (analogous to phonons).

ψ Entanglement Structure  
ψ supports entangled states across spatial regions. Reduced density matrix:

Plain text:  
ρA = TrB |Ψ⟩⟨Ψ|

Von Neumann entanglement entropy:

Plain text:  
SA = −TrA (ρA log ρA)

ψ fluctuations thus generate entanglement entropy scaling with region boundaries, suggesting holographic-like features.

ψ Vacuum Energy and Backreaction  
Vacuum contribution from ψ zero-point modes:

Plain text:  
ρvacψ = 1/2 ∫ d³k / (2π)³ ħ ωk

This modifies background ψ-curvature and contributes to effective cosmological terms.  
Renormalized vacuum energy density requires cutoffs or counterterms but may carry ψ-specific signatures.

ψ Correlation Length  
From propagator:

Plain text:  
G(r) ~ e^(−mψ r) / r

This defines correlation length ξ = 1/mψ.

* For massless ψ: long-range correlations.
* For massive ψ: short-range entanglement decay.

Simulation: ψ Vacuum Fluctuation Spectrum

# simulations/phase12\_part5\_vacuum\_fluctuations.py  
import numpy as np  
import matplotlib.pyplot as plt  
  
# Parameters  
hbar = 1.0  
mpsi = 1.0  
  
# Mode frequencies  
def omega(k, mpsi):  
 return np.sqrt(k\*\*2 + mpsi\*\*2)  
  
# Power spectrum of fluctuations  
def psi\_fluctuation\_spectrum(k, mpsi, hbar=1.0):  
 return hbar / (2 \* omega(k, mpsi))  
  
# Sample k range  
k\_vals = np.linspace(0.1, 5, 100)  
spectrum\_vals = [psi\_fluctuation\_spectrum(k, mpsi, hbar) for k in k\_vals]  
  
plt.plot(k\_vals, spectrum\_vals)  
plt.xlabel("Momentum k")  
plt.ylabel("⟨|ψ̃(k)|²⟩")  
plt.title("ψ Vacuum Fluctuation Spectrum")  
plt.grid(True)  
plt.show()